2d Physics and Beyond

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The aim of this short course is to provide the necessary background of Conformal Field Theory (CFT) for the further study of different topics in the following Math/Phys and Phys/Math directions, such as:

Math:
Infinite-dimensional algebras (Kac-Moody and Virasoro) and their representations.
Vertex operator algebras.
Boson-fermion correspondence.

Phys:
(Super) String theory and a lot of related things.
(Solvable) lattice models.

The lectures are destined for the BSc/MSc students (3-5 years of study and more) from the Mathematical Physics and High Energy Physics Departments of the Physics Faculty, St. Petersburg State University.

The overview of the course:

1. Basics from the classical field theory:

   Equations of motion. Noether currents. Energy momentum tensor. Field theory in curved spaces. Differential-geometrical background (Lie derivatives, Killing vectors, Symmetries (Diff-invariance as an important example)).

2. Weyl, scale and conformal invariance in D dimensions. Infinite-dimensional conformal symmetry in 2d. Global and Local transformations. Classical conformal invariance: conserved currents. Examples of invariant Lagrangians.

3. Towards quantum conformal invariance: Polyakov's bootstrap Program. The Operator Product Expansion (OPE).

4. Massless scalars in 2d: holomorphic ordering and examples of the OPE.

5. Conservaton of currents on the quantum level: Ward identities. Examples.

6. The cylinder and the plane: time and radial ordering. Correlation functions and quantum conformal invariance. Holomorphic currents and Contour-commutator trick. The mode expansion of energy-momentum tensor and Virasoro algebra. Central extension.

7. Primary fields and states. Massless scalars in 2d and the construction of primary fields. Descendant states.

8. Conformal vacuum. The adjoint operator. 2-, 3-point correlation functions. Zamolodchikov's inner product.

9. Oscillator mode expansion for the free massless scalars in 2d. Holomorphic and Fock ordering. The Fock space of states. The Virasoro generators in terms of oscillators.

10. Some facts about the representation theory of the Virasoro algebra. Unitarity. Singular vectors.

11. Singular vectors and the constraints on the correlation functions and the OPE. Minimal models and Rational CFT (RCFT) — a review. Lattice models.

12. Strings...Towards Strings.

For Exam: Exercises

1. Massless scalars on a circle. Space of states. The dependence on the radius of the circle.

2. Massless scalars with a dilaton. Energy-momentum tensor. Central charge. Primary fields.

3. b,c ghost system. Energy-momentum tensor. Central charge. Mode expansion. The construction of the conformal vacuum.

4. Free massless Majorana fermion. Energy-momentum tensor, OPE, central charge. R and NS sectors: mode expansion.

Recommended Books:

- P. Di Francesco, P. Mathieu, D. Senechal, «Conformal Field Theory» 1997, ch. 1-7

- J. Polchinski, «String Theory» 1998, ch. 2, 15